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CAN REMEDYING INDUCE CONCEPTUAL CHANGE IN STUDENTS' MISCONCEPTION OF LINEARITY?

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Abstract

Dalam kehidupan sehari-hari sering ditemukan masalah-masalah yang dapat dimodelkan ke dalam persamaan linear. Kenyataan ini menimbulkan persepsi dan generalisasi bahwa konsep linearitas dapat diaplikasikan terhadap semua permasalahan. Persepsi dan generalisasi ini tentu saja tidak benar. Generalisasi yang berlebihan dan persepsi ini seringkali ditemukan ketika siswa bekerja dengan konsep nonlinear, yakni siswa cenderung menggunakan konsep linearitas ketika menyelesaikan masalah nonlinear. Kesalahan seperti ini tentu saja bukan hanya kesalahan persepsi saja tetapi juga telah menjadi kesalahan konseptual. Banyak studi telah dilakukan guna mengetahui sumber kesalahan konseptual ini. Banyak pula studi yang dilakukan guna mengubah kesalahan konseptual pada siswa. Salah satu studi terkini dan sering diperbincangkan adalah studi yang dilakukan oleh Van Dooren, Hessel, Janssens, dan Verschaffel (2004). Dalam studinya, Van Dooren et.al (2004). melakukan intervensi terhadap 35 orang siswa kelas 8 di Finladers, Belgia. Terdapat beberapa aspek yang diabaikan dalam riset tersebut, dan berlawanan dengan konsep perubahan konseptual yang disampaikan. Berkaitan dengan masalah tersebut, pada artikel ini penulis akan mendiskusikan dan mengritisi studi yang dilakukan oleh Van Dooren et. al. (2004).

Keywords: *conceptual change, linearity problem, nonlinearity problem*

INTRODUCTION

Linear relationship is underlying model for approaching numerous practical and theoretical problem situations within mathematics and science (De Bock D. , Van Dooren, Janssens, & Verschaffel, 2007). We can find implementation of proportional concept almost in entire of mathematical building, from the idea of measuring magnitudes: length, area, and volume, the concept of ratio, the application of the "rule of three" in primary school to linear algebra and the use of linear models in calculus, and to the abstraction in a vector space sense in higher education (De Bock D. *et al.*, 2007). Since linear (or proportional) relation is often used in almost all of mathematics aspect of mathematics, it becomes a generalization that students consider that it is applicable in all of every problem or situation even in science

This repeated and continued applicability by every day experiences, likely to give confirmation about its validity that may cause a deeply entrenched misbelieve that every relationship between two quantities is linear (Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2004). Identifying the existence of linear misconception in the students earlier will be a good solution, however, the most important question is there any practical solutions to solve this problem or to change the students' conceptual of misconception of linear relation. Wim Van Dooren *et al*. in study conducted in 2004 purpose that remedying can be a good solution to this issue. In this article we discuss and criticise whether remedying done in the study intended can induce the conceptual change of students' illusion of linearity.

To well elaborate the discussion, we design the exploration of the study in these following flows. Firstly, we investigate the concept of linearity itself related to the problem posed in Wim Van Dooren *et al*. research, the cause of the occurrence of this issue. Secondly, we discuss a bit about conceptual change and the system of conceptual changing process. The discussion about research conducted by Wim Van Dooren *et al*. in 2004 will be the main topic of this essay. Finally, we close the discussion by answering the question posed in the thesis statement.

DISCUSSION

Misconception of Linearity

Many studies have been conducted towards the misconception of linear relation, also known as "illusion of linearity", "linearity trap", the "linearity obstacle", etc. (for review, see e.g, Behr, Harel, Post, & Lesh, 1992; Litwiller & Bright, 2002; Tournaire & Pullos, 1985; Verschaffel, De Corte, & Lasure, 1994; Verschaffel, De Corte, Lowyck, Dhert, & Vandeput, 2000; Verschaffel, Greer, & De Corte, 2000; Greer, 1993). The previous research conducted by De Bock D. , Van Dooren, Janssens, & Verschaffel (2002) found that students' improper use of linearity often result from superficial and intuitive reasoning, influenced by specific mathematical conception, habits and beliefs. Students' improper used leads to a deficient modelling process (De Bock D. *et al*., 2002).

The misuse of linear proportion problem can be found at different age levels and in a variety of mathematical domains (De Bock D. *et al*., 2002). The most frequent misconception of linear relation problem found concerns a geometry plane figure. The most famous and most often example quoted is Plato's dialogue *Meno* (see e.g., Berte, 1992, 1993; Daumas, 1989; Lelourad, Mira, & Nicolle, 1989; Rouche, 1989, 1992). When Plato asked the slave to double the area of a given square, the slave directly applies the idea of linear proportion by double the length of the square in order to get a double area. This problem was diagnosed and corrected after Socrates confronted the slave with a drawing. This example shows how Students strongly tend to see the linear relation between length and area or between length and volume as linear instead of quadratic and cubic, respectively. Consequently, students apply the linear scale factor in order to solve the problem instead doubling or cubing the length to determine the area or volume of an enlarged or reduced figure.

As the example of this phenomenon, we take a problem used in a study conducted by De Bock, *et al.* (2002); to paint 56 cm high Father Christmas drawing on the door of the bakery Bart need 6 ml paint. How much paint will approximately needed to paint the other Father Christmas drawing that is enlarged become 168 cm high? This study shows that based on this study's results, the tendency to use linear relation was strongly present in students' answer and reasoning.

There are some kinds of misconception of linearity that has been found and observed (for a review see, De Bock D. *et al.*, 2007):

- Students' improper application of linearity while solving arithmetic word problems.
- Over-reliance on linearity in graphical environments.
- Improper application of linearity in probabilistic situations
- Number patterns, algebra, and calculus
- The overuse of linearity in numerical estimation
- The misconception in case of geometrical problem

Related to the study conducted by Wim Van Dooren *et al*. (2004) the remedying activity intended that is going to be discussed in this essay is only limited in the misconception of geometrical problem.

Conceptual Changing

Although conceptual change has been discussed for several decades in the context of development research, science education research, and in the philosophy of science, the literature nevertheless presents a somewhat blurry picture of what exactly misconception are, what constitutes conceptual change, and why it is difficult (for a review see Chi & Roscoe, 2002 in Van Haaften, 2007). However, it seems that some researchers agree that human has their own conceptual understanding (see e.g. Mason, 2001; Visniadou, Ioannides, Dimitrakapoulou, & Papadementriou, 2001).

Human is the best creatures that have ever been created which equipped by thought. As human being, before studying in the school, children have their own knowledge and perception. They build their own conceptual structure and framework from everything they see, listen, and experience. This opinion is corresponding with socio-constructivist view on mathematics teaching and learning which say that students are not passive receivers of ready-made mathematics (see, e.g., Cobb, 1994; in Van Haaften, 2007). Instead of being a passive receiver, students are active learners who construct their mathematical thinking, tools and insight by relating what they experience in their environment. Thus, if they encounter new situations, they will relate the situation with their experience or cognitive schemata, and take their own conclusion based on this interaction. This important didactical process does not only involve the interaction between students and their environment but also their teachers.

Conceptual changing is a kind of systematic change in such forms of judgement (Van Haaften, 2007). To have an ability in judging whether it is a or b, students have to have a clear insight and perception about the new concept introduced. In order to get a clear perception, students need teacher roles.

Changing students' conceptual understanding and paradigm is including activity of changing their belief, habits even their intuitive reasoning. Thus rather than simple mechanism, many researchers agree to say that it is a complex mechanism process. Clear empirical indications for the intuitive application of linearity are found in the in-depth investigation reported in study conducted by De Bock *et al*. (2002). Based on the results of this study, students opted very quickly - almost immediately after confrontation with the problem – for a proportional solution method. In the analysis of literature, we elaborate the analysis with some arguments about conceptual changing and intuition point of view.

Analysis from Literature

The discussion in this article focuses on the study conducted by Wim Van Dooren *et al*. in 2004. The goal of this study was to break students' deep-rooted tendency in applying linear strategies in non-linear situations by giving remedying. To be more specifically, this study focused in breaking students' misconception to the context of the relationship between the linear measures of a figure and its perimeter, area, and volume.

This study involved 35 of 8 graders students in Flanders (Belgium). These students are grouped into different classes which both of them have same number of boys and girls. During two-week period of research, the experimental group (18 students) got test before intervention (pre-test), another test after intervention (post-test) and retention test, while a control group (17 students) only got a pre-test and retention test without any intervention process. Because the control group do not have intervention, then they are not expected to make a progress in the retention test. The intervention is conducted in 10 developed experimental lessons.

Lesson 1 and 2 focused on how to help student to understand the concept of similarity itself. In these teaching activities students not only learned to recognize and construct similarly enlarged and reduced figures and objects but also understand their properties.

The central topic of lesson 3 was the linear relations in similar figures. The goal of this activity was to make students' understanding of the proportional relations explicit, to show how this knowledge applies to similar figures and how it can be used to predict length in other size of figure.

The ideas of linear growth of perimeter and the quadratic growth of area were introduced and explored in the lesson 4 and 5. These lessons pursued students' understanding and ability to apply the principle that if a figure is enlarged or reduced k times, the perimeter of that figure enlarges k times too, but the area enlarges k^2 times.

The cubic growth of volume was the main discussion in the 6. This lesson pursued students' understanding and ability to apply the principle that if a figure is enlarged or reduced k times, the volume of that solid enlarges k^3 times (and its surface area enlarges k^2 times).

In lesson 8 to 10, the teaching experiments are designed to let students conclude an integrative project on the life and work of gnomes (Poortvliet & Huygen, 1977).

These structured lesson are designed in attempting to break students' intuitive use of linearity by thoroughly discussing the applicability of a (proportional) model for given situation. In conclusion, the problems that were given to students consist of proportional items and non-proportional items. All of the problems given were open-ended questions, thus students had to write down their answers which were scored either as correct or incorrect. The incorrect answers were categorized based on an analysis of the students' solution steps.

The pre-test results showed that there were no significant differences between two groups. Both of the groups' pre-test result shows that both of group tend to have the same problem. It is indicated that the initial starting points are the same and both of the groups considered to be compared.

After having intervention, the experimental group significantly improved on non-proportional item, even though the result showed was not as highly expected. The control group result also improves slightly even though the improvement is not as much as the experimental group's. According to the analysis of students' results in post-test there was significant increase on the proportional items from pre-test. The experimental group result then had non-significant decrease in the performance on the retention test.

However, the analysis of the student answers show that instead of getting well understanding, after intervention many of students were confused and made new conceptual and technical error in applying non—linear solution on non-linear problems than the control group's results (Van Dooren, *et al*., 2004).

Based on this result, it seems that instead of giving a new perspective toward the problem, the new concept (non-linear proportion) replace the initial view of students' understanding or concept to linear proportion. This is what Mortimer (1995) judge as an inappropriate conceptual changing system. As Chi (1991, in Mortimer 1995) showed that it is possible to coexistence of two concepts for the appropriate context, we would argue that students have to know how linear and non-linear problem are coexistent by experience, arguing, criticism, and being confronted.

By analysing the Van Dooren *et al*. study, we find the fact that this study done in really short period. By tagging the idea of changing students conceptual understanding, two weeks period of times is too shorts to make it happens. This judgement made by considering that changing students' conceptual understanding is not as simply as giving a structured lesson. There is a complicated system behind someone else conceptual understanding.

Even if Van Dooren *et al.* (2004) argue that this study as a *re-subsumption* of students conceptual changing from using linear proportion to non-linear proportion, we still cannot see that the treatments given to student showing so. According to Ohsslon's theory of conceptual change *re-subsumption* (see in Chinn & Samarapungavan, 2009) conceptual change can occurs when a person uses analogical transfer to map the conceptual system of non-linear proportion from linear proportion. Rather than doing *re-subsumption*, what we see in this study is just like the bombarded information that non-linear proportion replaces the linear proportion's function. This judgement is made considering a lot of different topics that should be discuss within short period of time.

Learning is a process when students construct their understanding by criticism, arguing, revising, reconstructing and re-organizing their understanding. However, in this lesson series, students are only confronted with the invalidity of their conceptual understanding rather than giving analogical transfer to map students' conceptual understanding.

Furthermore, in these lesson series students do not have opportunity to discuss with their friends in order to sharpen their understanding, arguing each other's or even criticism every problem served. It means that students do not have opportunity to elaborate the topic deeper. Changing students' conceptual understanding means changing students' paradigm in seeing linear proportion problem into non-linear proportional problem. As has been mentioned in the misconception of linearity, the misconception of linearity is a result from superficial and intuitive reasoning, influenced by specific mathematical conception, habits and beliefs. Since students often find that they can rely on linearity in some specific mathematical conception mentioned, it leads students to have it in their logical reasoning.

According to De Bock D. (2007) it has often been argued that when the linearity concept is being developed, it tends to receive a more and more self-evident, even intuitive status. As often as they use the linear proportion in their logical reasoning, it is realised or not this issue will be deep rooted implanted in their thinking, habits, and belief. This repeated logical reasoning leads students over-believe even become as a matter of intuition. When it becomes intuitive reasoning, this issue becomes more difficult to be fixed especially if we only do the remedying in short time period, because intuitions are resistant to change. Intuition is profoundly related to our adaptive system, which is shaped by early and repeated experiences (De Bock D., *et al*, 2007).

Furthermore, based on De Bock D., *et al.* (2007), in order to change students conceptual understanding by overcoming the intuition reasoning, and to change the students' deep rooted habits and beliefs contributing to superficial modelling process, a teaching intervention during a longer time remedying period or spread out the retention activities over a longer time period are necessary. This statement contradicts with the fact that Van Dooren et al. (2004) study is conducted within two weeks only. Thus, two weeks period intervention of ten lessons elaborated by Van Dooren et al. remains fragile and unstable. Elaborating the lesson in short period of intervention feared would create a new problem. This anxiety is proven by students' result which indicates that after having this remedying activity, students tend to over-generalized of non-linear problems. This short period of time-intervention even make students have serious difficulties in knowing what concept should be used in certain context or problem of situation.

Instead of giving a short period of retention or remedying process, we suggest teacher - who have a big role in the class room - to do some concrete educational practical way. There are many other ways can be done to induce conceptual change in students misconception in linearity. Since this issue involve intuition, belief and habits, these following effort can be a better solution against intuition, belief and habits that are constructed by repeated experience rather than giving a short period of intervention. These following concrete educational practical ways have been made by Verschaffel, Greer, and De Corte (2002):

- Break up the expectation that any word problem can be solve by adding, subtracting, multiplying, or dividing, or by a simple combination thereof.
- Eliminate the flaws in textbooks that allow superficial solution strategies to be undeservedly successful.
- Vary problem so that it cannot be assumed that all data included the problem, and only those data, are required for the solution.
- Weed out word problems in which situation, the numbers, and/or the question do not correspond to real life or for which the mathematical model that students are expected to find and apply does not fit (well) with the situation evoked by the problem statement.
- Legitimize forms of the answer other than exact numerical answers, e.g. estimations, commentaries, drawings, graphs, etc.

- Include alternative forms of tasks such as classification task and problem-posing task.

CONCLUSION

The result of study conducted by Van Dooren *et al*. (2004) showed that both group i.e. experimental group and control group, experienced improvement in their results. The experimental group results increase significantly, and surprisingly the control group that was not expected to have an improvement also experience increase even though it was not as much as the experimental group.

Even though the experimental group shows a significant improvement in their result compared to the control group, their answers' analysis reveals that they are confused of linear and non-linear proportion. It reveals that the intended conceptual change was not fully accomplished, since students still experienced serious difficulties in choosing the appropriate context to use. Moreover, instead of having good understanding as purposed in this research, many students did not develop deeper understanding and a disposition to distinguish problem that can be solved by linear proportional relation and non-linear proportion, after the remedying process (Van Dooren, *et al.,* 2004).

Based on this found, we can conclude that this remedying activity is remained fragile and unstable since it is elaborated in short time period and restricted instructional setting. Thus, this remedying activity cannot be fully considered to help student in the conceptual change of students' illusion of mathematics although the results show a significant improvement.

Learning is a complex activity in reconstructing students understanding involving some activities i.e. criticism, arguing, revising, reconstructing and re-organizing their understanding. Thus, to have the intended conceptual changing - in order to break deep-rooted of misconception of linearity - that is involved the matter of intuitive, beliefs, and habits, a longer time remedying period or spread out the retention of activities over a longer time period are necessary.

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